Edge Local Differential Privacy for Dynamic Graphs[1]

Presented by
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Outline

Introduction

- Huge amount of data is generated every day in networked systems.
- Nevertheless, in reality, nearly all networks undergo changes, with nodes or edges arriving or going away as the system develops.
- Therefore, static graph networks are not adequate to model these kinds of network structures.

Dynamic Graph

- Data on dynamic networks here is a collection of successively obtained, equally spaced snapshots of the network topology
- These snapshots are a set of different networks defined on the same set of nodes.
- A dynamic network graph model consists of an initial state G_0 and states G_i, for $i = 1, \ldots T$, defined by:

$$G_i = \mathcal{A}_{1-\alpha,1-\beta}(G_{i-1})$$

We will denote it as: $G(G_0, T, \alpha, \beta)$.

Challenges of Privacy in Dynamic Graph

- The adversaries can use their information about the structural graph to infer private information from the graph.
- Proper privacy models have been developed for static graphs following k-anonymity and differential privacy.

Motivation

- The extension of the definition of local differential privacy for edges to dynamic graphs;
- The privacy mechanisms for providing graphs compliant with edge-local differential privacy for dynamic graphs. This is achieved by applying the noise-graph mechanism;

Noise Graph Mechanism

For any graph G with n nodes, and two probabilities p_0 and p_1 We define the following noise-graph mechanism:

$$\mathcal{A}_{p_0,p_1}(G) = G \oplus G_0 \oplus G_1,$$

Such that:

$$G_0 = G' \setminus G \text{ for } G' \in \mathcal{G}(n, 1-p_0)$$
$$G_1 = G'' \cap G \text{ for } G'' \in \mathcal{G}(n, 1-p_1).$$

Stochastic matrix associated to the noise graph

The probabilities of randomization of an edge or a non-edge in a graph G after applying the noise-graph mechanism $\mathcal{A}_{p_0,p_1}$ are represented by the following stochastic matrix:

$$P = P(\mathcal{A}_{p_0,p_1}) = \begin{pmatrix} p_0 & 1-p_0 \\ 1-p_1 & p_1 \end{pmatrix}$$

Dynamic-network-graph-model

The dynamic network graph model consists of an initial state G_0 and states G_t, for $t = 1, \ldots T$, defined by:

$$G_t = \mathcal{A}_{1-\alpha,1-\beta}(G_{t-1})$$

We will denote it as: $G(G_0, T, \alpha, \beta)$.

Local differential privacy

A randomized algorithm π, satisfies ε-local differential privacy if for all inputs x, x' and all outputs $y \in Range(\pi)$:

$$P(\pi(x) = y) \leq e^{\varepsilon} P(\pi(x') = y) \tag{1}$$

Edge Differential Privacy

- A randomized algorithm $\mathcal{A}$ satisfies ε-edge local differential privacy if for all pairs of nodes u, v, all times stamps t and edge values i, j, k:

$$Pr[\mathcal{A}(u, v, t; i) = k] \leq e^{\varepsilon} Pr[\mathcal{A}(u, v, t; j) = k]$$

  we say that $\mathcal{A}$ is ε-edge locally differentially private (ε-eLDP).

Algorithm 1: Dynamic Network mechanism

Let $G = G_0, \ldots G_T$ be a dynamic graph. We define the dynamic network mechanism as:

$$\mathcal{D}_{p_0,p_1}(G) = G(g_0, T, 1 - p_0, 1 - p_1),$$

where, $g_0 = \mathcal{A}_{p_0,p_1}(G_0)$. That is, the protected dynamic graph $g_0, g_1, \ldots, g_T$ corresponds to

$$g_i = \mathcal{A}^{i+1}_{p_0,p_1}(G_0).$$

Algorithm 2: Parallel Protection Mechanism

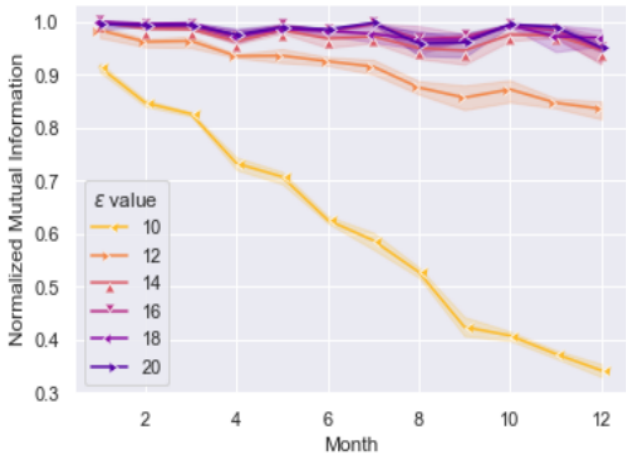
Let $G = G_0, G_1, \ldots, G_T$ be a dynamic graph. Let $\mathcal{A}_{p_0,p_1}$ denote the noise-graph mechanism. Then, we define the parallel protection of the dynamic graph with parameters p_0 and p_1 as the protection process that provides $\tilde{G} = \tilde{G}_0, \tilde{G}_1, \ldots, \tilde{G}_T$ with $\tilde{G}_i = \mathcal{A}_{p_0,p_1}(G_i)$ for $i = 0, \ldots, T$. We denote the parallel protection of a dynamic graph G with parameters p_0 and p_1 as $\mathcal{A}^{||}_{p_0,p_1}(G)$.

Experiment and Results I

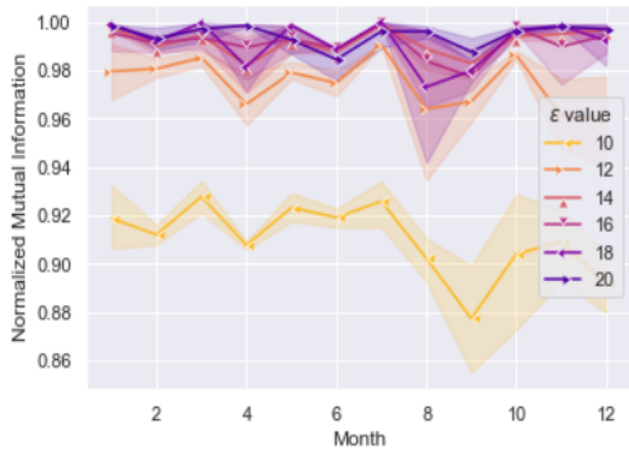
Table 1: Preprocessed datasets statistics

Dataset	No. of nodes	No. of Edges	Avg. Snapshot Density
CAIDA-AS	5,715	403,761	0.0010
DBLP	25,439	450,878	0.00007

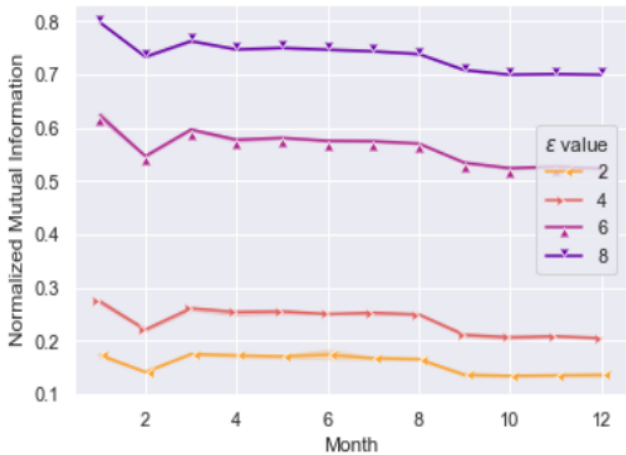
Experiment and Results (NMI vs Month for CAIDA-AS data)



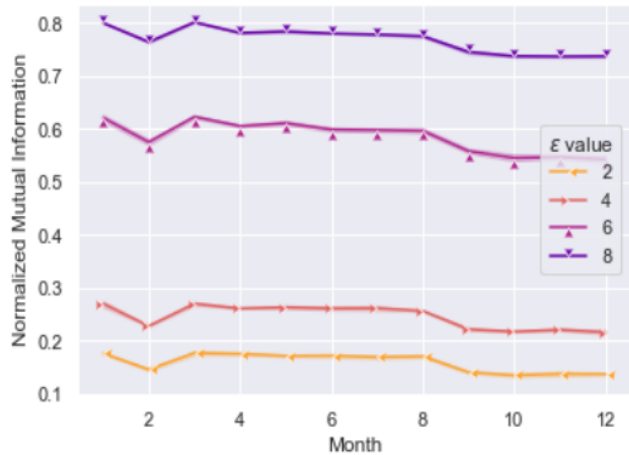
(a) Dynamic mechanism for large ε values



(b) Parallel mechanism for large ε values

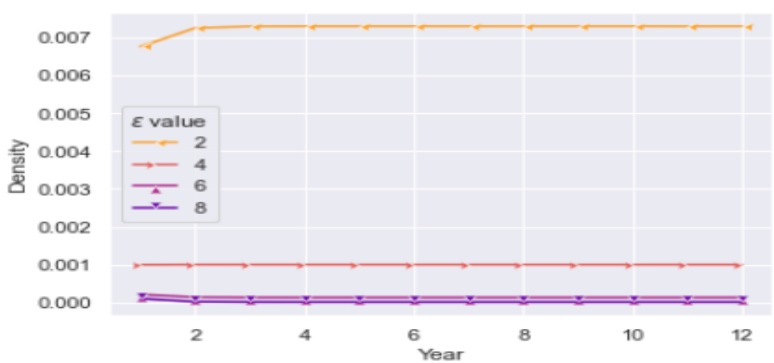


(c) Dynamic mechanism for small ε values

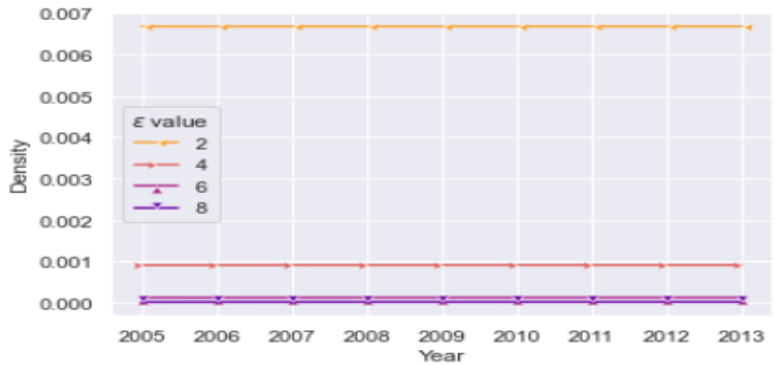


(d) Parallel mechanism for small ε values

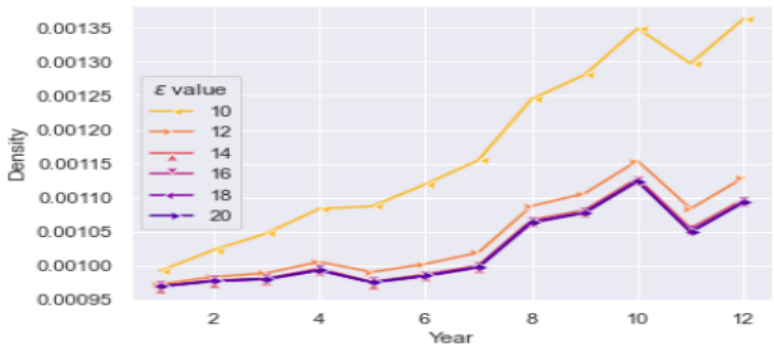
Experiment and Results III



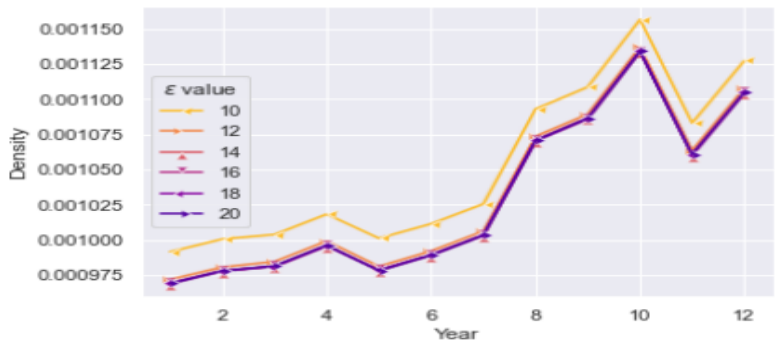
(a) Dynamic mechanism for small ε values



(b) Parallel mechanism for small ε values



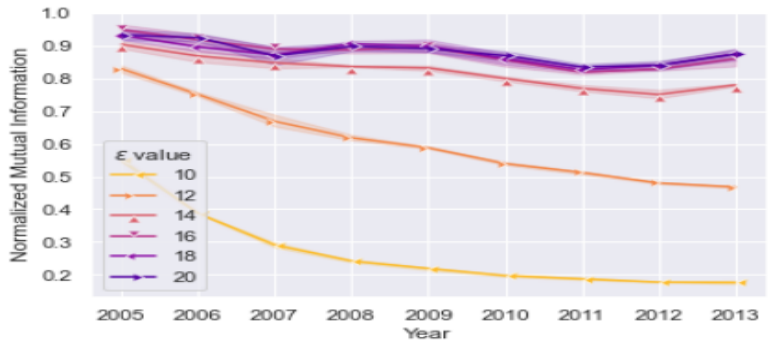
(c) Dynamic mechanism for large ε values



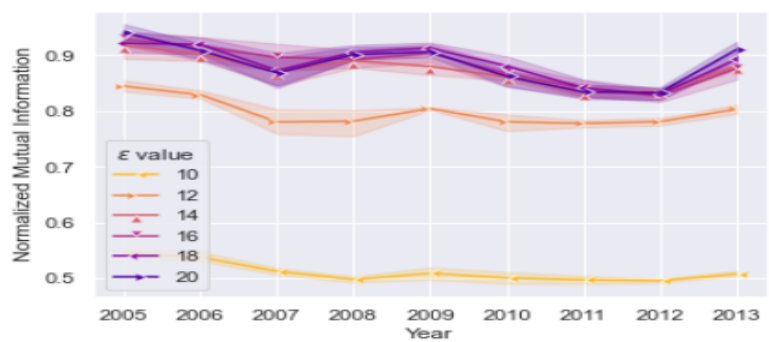
(d) Parallel mechanism for large ε values

Fig. 2: Densities for the snapshot-graphs obtained by applying the dynamic and parallel mechanisms to CAIDA-AS.

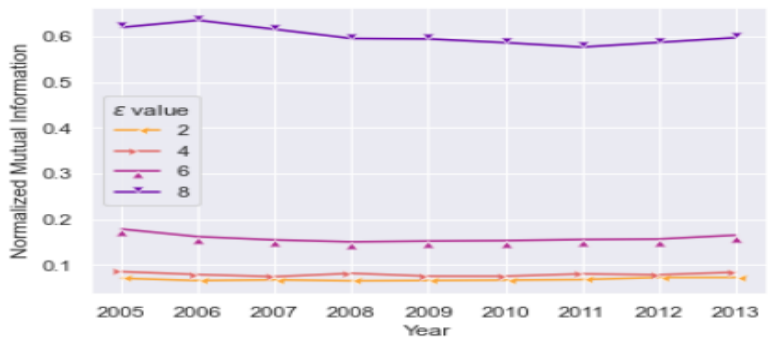
Experiment and Results IV



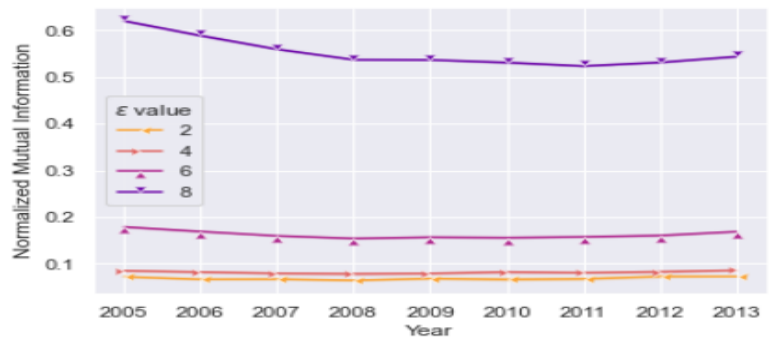
(a) Dynamic mechanism for large ε values



(b) Parallel mechanism for large ε values



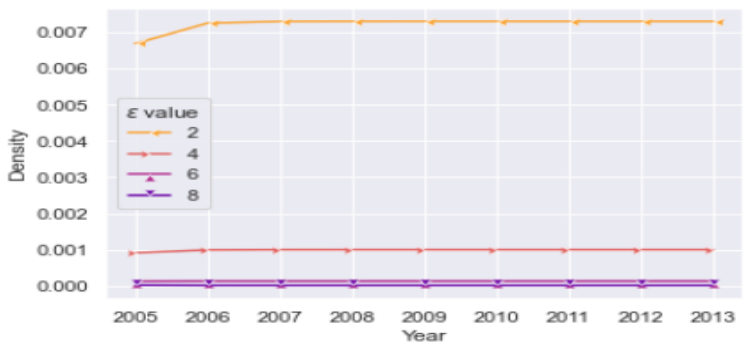
(c) Dynamic mechanism for small ε values



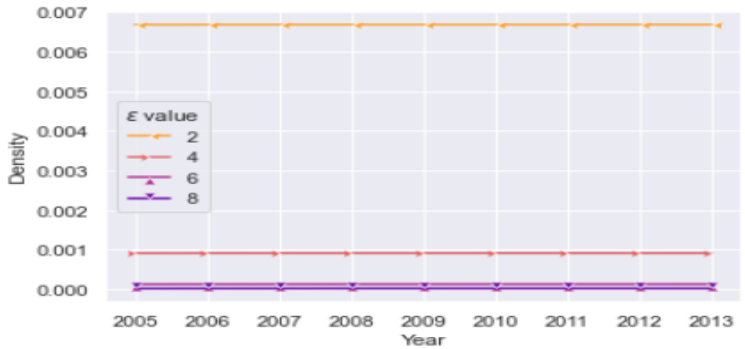
(d) Parallel mechanism for small ε values

Fig. 3: Normalized mutual information between the communities detected on the DBLP data and the data protected with the dynamic and parallel mechanisms for several ε values.

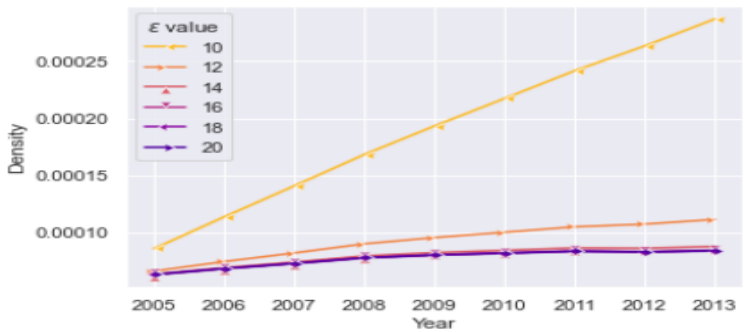
Experiment and Results V



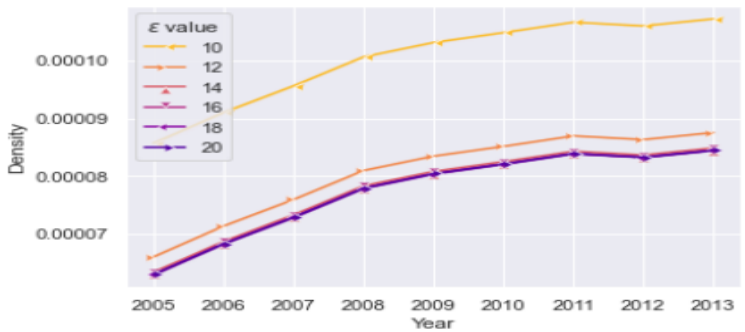
(a) Dynamic mechanism for small ε values



(b) Parallel mechanism for small ε values



(c) Dynamic mechanism for large ε values



(d) Parallel mechanism for large ε values

Fig. 4: Densities for the snapshot-graphs obtained by applying the dynamic and parallel mechanisms to DBLP.

Thank You!!