

# Edge Local Differential Privacy for Dynamic Graphs<sup>1</sup>

Presented by  
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# Outline

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# Introduction

- | Huge amount of data is generated every day in networked systems.
- | Nevertheless, in reality, nearly all networks undergo changes, with nodes or edges arriving or going away as the system develops.
- | Therefore, static graph networks are not adequate to model these kinds of network structures.

# Dynamic Graph

- | Data on dynamic networks here is a collection of successively obtained, equally spaced snapshots of the network topology
- | These snapshots are a set of different networks defined on the same set of nodes.
- | A dynamic network graph model consists of an initial state  $G_0$  and states  $G_i$ , for  $i = 1; \dots; T$ , defined by:

$$G_i = A_{1 \dots i} (G_{i-1})$$

We will denote it as:  $G(G_0; T; \dots)$ .

# Challenges of Privacy in Dynamic Graph

- | The adversaries can use their information about the structural graph to infer private information from the graph.
- | Proper privacy models have been developed for static graphs following  $k$ -anonymity and differential privacy.

# Motivation

- | The extension of the definition of local differential privacy for edges to dynamic graphs;
- | The privacy mechanisms for providing graphs compliant with edge-local differential privacy for dynamic graphs. This is achieved by applying the noise-graph mechanism;

# Noise Graph Mechanism

For any graph  $G$  with  $n$  nodes, and two probabilities  $p_0$  and  $p_1$  We define the following noise-graph mechanism:

$$A_{p_0; p_1}(G) = G \quad G_0 \quad G_1;$$

Such that:

$$G_0 = G^0 \cap G \text{ for } G^0 \geq G(n; 1 - p_0)$$

$$G_1 = G^{00} \setminus G \text{ for } G^{00} \geq G(n; 1 - p_1):$$

# Stochastic matrix associated to the noise graph

The probabilities of randomization of an edge or a non-edge in a graph  $G$  after applying the noise-graph mechanism  $\mathcal{A}_{\rho_0; \rho_1}$  are represented by the following stochastic matrix:

$$P = P(\mathcal{A}_{\rho_0; \rho_1}) = \begin{pmatrix} \rho_0 & 1 & \rho_0 \\ 1 & \rho_1 & \rho_1 \end{pmatrix}$$



# Dynamic-network-graph-model

The dynamic network graph model consists of an initial state  $G_0$  and states  $G_t$ , for  $t = 1; \dots; T$ , defined by:

$$G_t = A_{1 \dots t} (G_{t-1})$$

We will denote it as:  $G(G_0; T; \dots)$ .

# Local differential privacy

A randomized algorithm  $\mathcal{A}$ , satisfies  $\epsilon$ -local differential privacy if for all inputs  $x, x^0$  and all outputs  $y \in \text{Range}(\mathcal{A})$ :

$$P(\mathcal{A}(x) = y) \leq e^\epsilon P(\mathcal{A}(x^0) = y) \quad (1)$$

# Edge Differential Privacy

- | A randomized algorithm  $A$  satisfies  $\epsilon$ -edge local differential privacy if for all pairs of nodes  $u; v$ , all times stamps  $t$  and edge values  $i; j; k$ :

$$\Pr[A(u; v; t; i) = k] \leq e^\epsilon \Pr[A(u; v; t; j) = k]$$

we say that  $A$  is  $\epsilon$ -edge locally differentially private ( $\epsilon$ -eLDP).

# Algorithm 1: Dynamic Network mechanism

Let  $G = G_0; \dots; G_T$  be a dynamic graph. We define the dynamic network mechanism as:

$$D_{\rho_0; \rho_1}(G) = G(g_0; T; 1 \quad \rho_0; 1 \quad \rho_1);$$

where,  $g_0 = A_{\rho_0; \rho_1}(G_0)$ . That is, the protected dynamic graph  $g_0; g_1; \dots; g_T$  corresponds to

$$g_i = A_{\rho_0; \rho_1}^{i+1}(G_0):$$

## Algorithm 2: Parallel Protection Mechanism

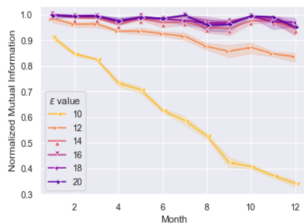
Let  $G = G_0; G_1; \dots; G_T$  be a dynamic graph. Let  $A_{\rho_0; \rho_1}$  denote the noise-graph mechanism. Then, we define the parallel protection of the dynamic graph with parameters  $\rho_0$  and  $\rho_1$  as the protection process that provides  $\mathcal{G} = \mathcal{G}_0; \mathcal{G}_1; \dots; \mathcal{G}_T$  with  $\mathcal{G}_i = A_{\rho_0; \rho_1}(G_i)$  for  $i = 0; \dots; T$ . We denote the parallel protection of a dynamic graph  $G$  with parameters  $\rho_0$  and  $\rho_1$  as  $A_{\rho_0; \rho_1}^{jj}(G)$ .

# Experiment and Results I

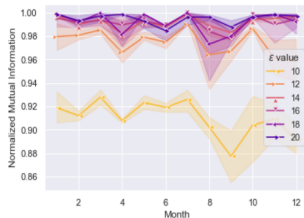
Table 1: Preprocessed datasets statistics

Dataset	No. of nodes	No. of Edges	Avg. Snapshot Density
CAIDA-AS	5,715	403,761	0.0010
DBLP	25,439	450,878	0.00007

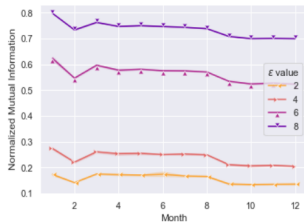
# Experiment and Results (NMI vs Month for CAIDA-AS data)



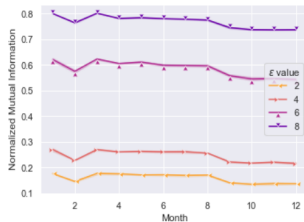
(a) Dynamic mechanism for large  $\epsilon$  values



(b) Parallel mechanism for large  $\epsilon$  values

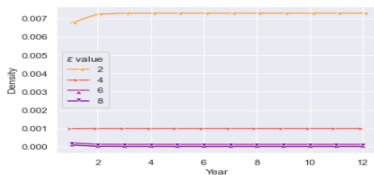


(c) Dynamic mechanism for small  $\epsilon$  values

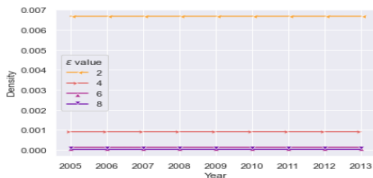


(d) Parallel mechanism for small  $\epsilon$  values

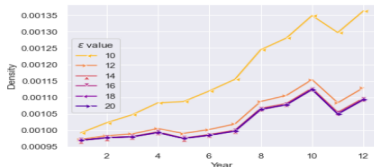
# Experiment and Results III



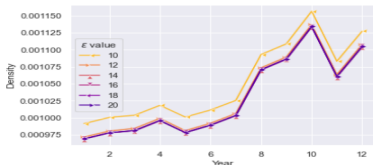
(a) Dynamic mechanism for small  $\varepsilon$  values



(b) Parallel mechanism for small  $\varepsilon$  values



(c) Dynamic mechanism for large  $\varepsilon$  values

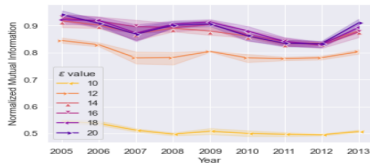
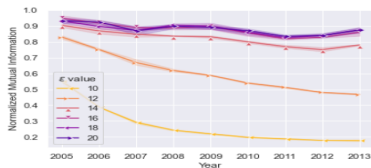


(d) Parallel mechanism for large  $\varepsilon$  values

Fig. 2: Densities for the snapshot-graphs obtained by applying the dynamic and parallel mechanisms to CAIDA-AS.

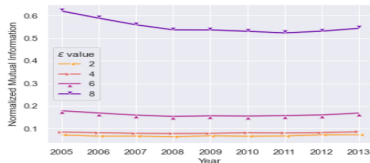
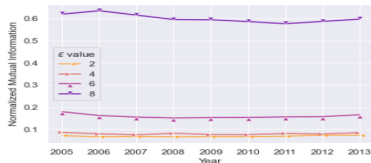


# Experiment and Results IV



(a) Dynamic mechanism for large  $\epsilon$  values

(b) Parallel mechanism for large  $\epsilon$  values

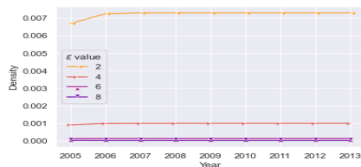


(c) Dynamic mechanism for small  $\epsilon$  values

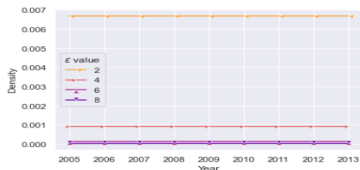
(d) Parallel mechanism for small  $\epsilon$  values

Fig. 3: Normalized mutual information between the communities detected on the DBLP data and the data protected with the dynamic and parallel mechanisms for several  $\epsilon$  values.

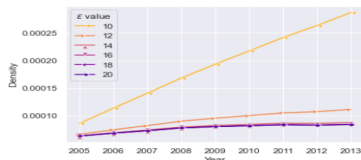
# Experiment and Results V



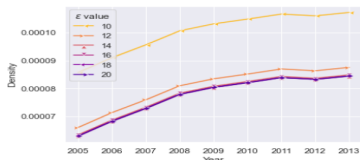
(a) Dynamic mechanism for small  $\epsilon$  values



(b) Parallel mechanism for small  $\epsilon$  values



(c) Dynamic mechanism for large  $\epsilon$  values



(d) Parallel mechanism for large  $\epsilon$  values

Fig. 4: Densities for the snapshot-graphs obtained by applying the dynamic and parallel mechanisms to DBLP.

Thank You!!